Computer Vision-Optical Flow

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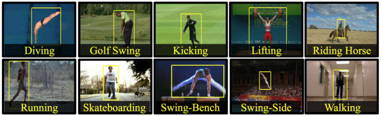
Overview

1. What is Computer vision?
2. Optical Flow Estimation
3. Brightness Constraint
4. Aperture Problem
5. KLT Algorithm
6. Condition for solvability
7. Eigenvalue interpretation
8. Applications
9. Interesting Examples

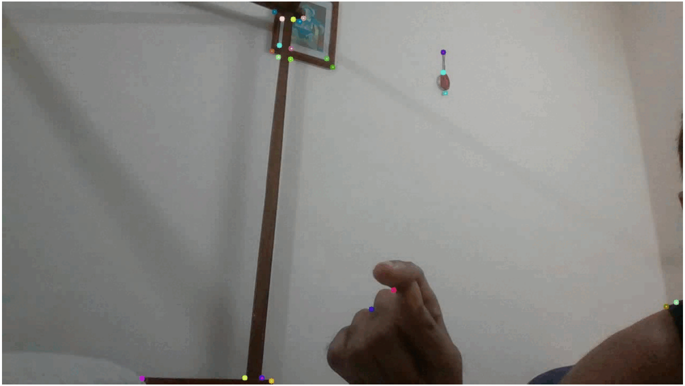
- Computer vision is the field of computer science and Artificial Intelligence that deals with replicating complex functionalities of our human eye and help computers perceive and process the images/videos in the same way.

(a) Image segmentation(Source)



(b) Activity Recognition(Source)

Optical Flow Example



Estimating Optical Flow

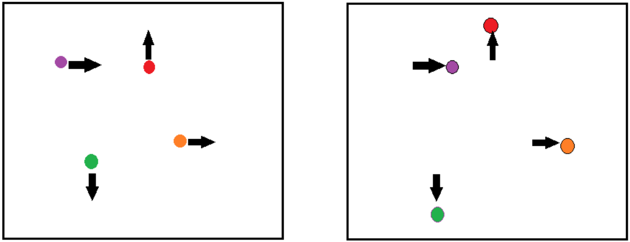
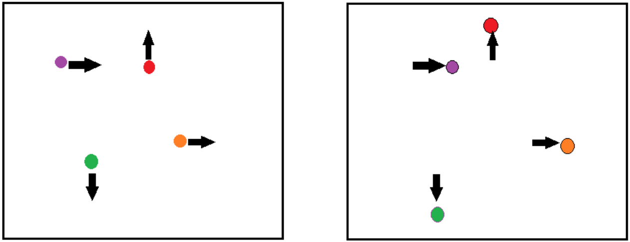


Figure 3: Transition from I(x,y,t) to I(x,y,t+1)

Three important assumption for estimating optical flow-

- Brightness Constancy

Estimating Optical Flow



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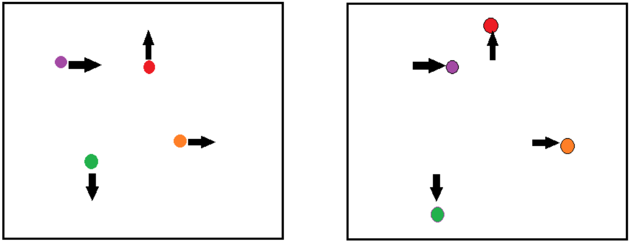

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Three important assumption for estimating optical flow-

- Brightness Constancy
- Small motion
- Spatial coherence

Brightness Constancy Constraint

1. **Brightness Constraint**

$$I(x, y, t-1) = I(x + u(x, y), y + v(x, y), t)$$

$$\textit{Linearizing using Taylor's series expansion}$$

$$I(x+u, y+v, t) \approx I(x, y, t-1) + I_x u(x, y) + I_y v(x, y) + I_t$$

$$I(x+u, y+v, t) - I(x, y, t-1) \approx I_x u(x, y) + I_y v(x, y) + I_t$$

$$\nabla I [u \ v]^T + I_t \approx 0$$

Aperture Problem

1. gradient constraint provides 1 constraint in 2 unknowns u,v
2. gradient constrains the velocity in normal direction
3. $u_n = -\frac{f_t}{\|\vec{\nabla} f\|}\frac{\vec{\nabla} f}{\|\vec{\nabla} f\|}$;If $\vec{\nabla} f = 0$ then normal velocity is undefined hence we get no constraint.

Figure 4: Horizontal Edge detects vertical motion

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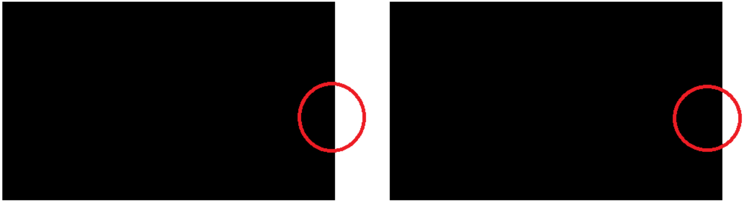

Figure 5: Vertical Edge detects horizontal motion

Aperture Problem

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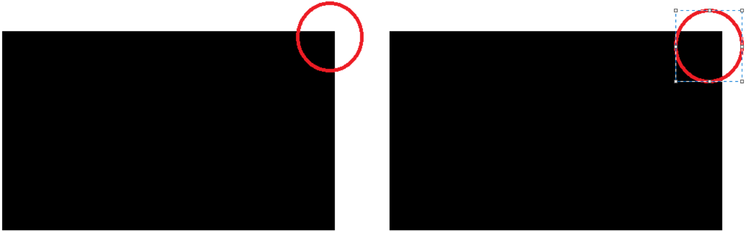

Figure 6: Corners detect both

Lucas Kanade Object Tracking Algorithm

$$I_t(p_i) + \nabla I(p_i).[u\ v] = 0$$

$$\begin{bmatrix} I_x(p1) & I_y(p1) \\ I_x(p2) & I_y(p2) \\ . & . \\ . & . \\ I_x(p25) & I_y(p25) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(p1) \\ I_t(p2) \\ . \\ . \\ I_t(p25) \end{bmatrix}$$

$$A\ d = b$$

$$Least\ squares\ solution\ for\ d\ is\ given\ by\ (A^T A)d = A^T b$$

$$\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$

$$A^T A \qquad\qquad\qquad\qquad A^T b$$

Conditions for solvability

The following above equation is solvable uder the given conditions-

- $A^T A$ should be invertible

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- $A^T A$ should be invertible
- Eigen values λ_1 and λ_2 should not be too small in magnitude
- $A^T A$ should be well conditioned i.e Eigen value $\frac{\lambda_1}{\lambda_2}$ should not be too large and λ_1 being the larger of them.

The Matrix $M = A^T A$ *is the Corner detection matrix* !!

Corner detection & Eigenvalue Interpretation

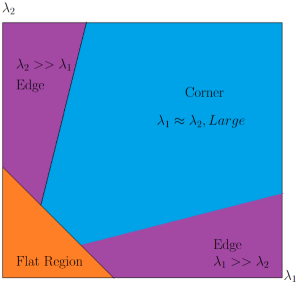

Figure 1: Eigen value interpretation

(a) $\lambda_1\lambda_2 - k(\lambda_1 + \lambda_2)^2$

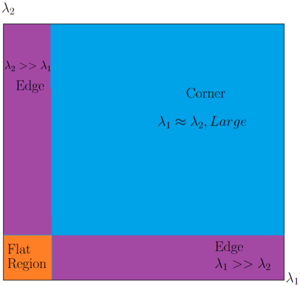

Figure 1: Eigen value interpretation-Shi Tomasi

(b) $min(\lambda_1, \lambda_2)$

Temporal Aliasing Coarse to fine Gaussian Pyramids

Applications-Precipitation Nowcasting

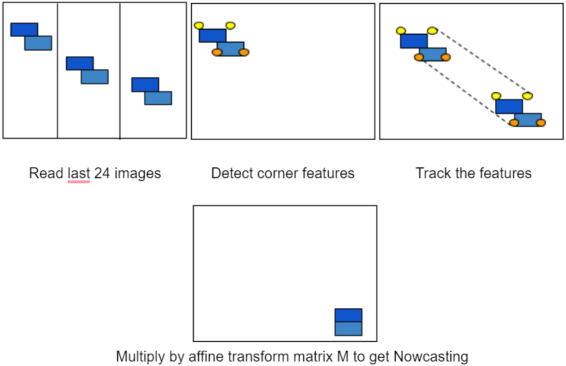

Figure 8: Steps to deduce rainfall in near future using Optical flow

Errors in Lucas Kanade Algorithm

(Source)

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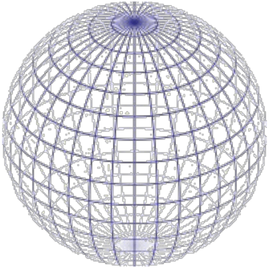

(Source)

Optical Flow without Motion !!

Figure 9: Optical Motion (Source)

Thank You